Thermoacoustic Instabilities in a Rijke Tube with Heating and Cooling Elements

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Abstract
Thermoacoustic instabilities are investigated in a Rijke tube with both heating and cooling elements. The effects of the time delays of the elements, magnitudes of the temperature jumps, and location and compactness of the cooling element are investigated. The results show that for small temperature jumps, the system behaves periodically with changing source time delay. However, for larger temperature jumps corresponding to realistic combustion systems, various stable and unstable situations can occur, which simultaneously depend on the source and sink time delays. This dependence in some configurations is to the extent that the system is predicted unstable or stable, depending on whether or not the heat exchanger is included. In addition, the location or compactness of the heat sink has opposing effects depending on the source time delay. This investigation shows that a correct prediction of the system stability requires taking into account the coupled effects of the heat source and sink. An extension of this verified model can be used to predict instabilities in full-scale domestic heating systems.

Introduction
Thermoacoustic instabilities are usually associated with lean (partially) premixed combustion systems and have been studied for more than a century [1–4]. Such systems have various implications, such as gas turbines, boilers and other heating systems. Many of such systems include heat sources (burners) and heat sinks (heat exchangers). Studying thermoacoustics in such systems is usually nontrivial due to the inherent coupling of hydrodynamics, chemical reactions and acoustics. One common practice is to treat the burner as the active thermoacoustic element and the other elements as passive elements (such as ducts, vessels, terminations and heat exchangers) [5, 6]. One of the simplest and yet interesting configurations to study thermoacoustic instabilities in such systems is the Rijke tube, which shows the coupling of a heat source with the acoustics in a simple cavity (tube with two open ends). This configuration is in many cases a good representation of unstable practical combustion systems [7, 8]. There are various studies available on the coupling of flames and acoustic waves. Consequently, many researchers have investigated various types of heat sources in combustion systems that actually include heat exchangers in reality [9, 10]. On the other hand, the transient heat transfer phenomenon in an acoustically forced flow is well studied in the past [11–13]. However, the possible role of the heat exchanger as an active acoustic element in combustion systems has mostly been ignored.

Recently, some researchers have investigated the simultaneous coupling between acoustics and both heat sources and sinks. However, these studies are limited to specific cases [14, 15] or calculation of transfer functions without discussing the stability limits of the system [16]. In addition, there are studies performed on Rijke tubes with multiple heating elements with the focus on active or passive control of the instabilities [17, 18]. This approach lacks practicality, because in practical combustion systems, the post combustion temperature is high, and creating a second temperature jump in the system requires large power. On the other hand, the heat exchanger is an existing element in many combustion systems and it is worthwhile to investigate if it can be used as an extra parameter in the thermoacoustic design of such systems.

For the current work, we investigate a Rijke tube model including both heating and cooling elements. This configuration helps revealing the combined effects of heating and cooling on the thermoacoustics in heating appliances. The ultimate goal is to answer questions such as, how can the design of the heat exchanger affect the thermoacoustic behavior of the system?

We perform these investigations in multiple steps of adding complexity in order to perform a detailed analysis of the nontrivial problem. First, we consider the Rijke tube with only one acoustically active heat source and a positive temperature jump. Then, we add a similar heat sink with a reverse temperature jump, and vary the magnitude of these temperature jumps as well as the time delays of the source and sink. This part of the investigation is motivated by the fact that the temperature jump and time delay of the heat source is immediately related to the combustion properties of the burner in heating appliances. A leaner flame results in larger flame time delays and smaller temperature jumps across the burner [19–21]. In addition, the use of non-conventional fuels can also result in various combinations of flame time delay and temperature jump.

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The next step involves investigating a heat sink with distributed elements and changing its location and compactness in the tube. The distance between the heat source and sink is a key parameter in designing the emission properties of heating appliances through creating a compromise between NOx and CO emissions. In addition, the heat sink with single (compact) and multiple (distributed) elements are representative of two major heat exchanger designs in such systems, i.e. compact tubes vs. distributed pins.

The results show that various stable and unstable situations can occur, which simultaneously depend on the source and sink time delays. Finally, we conclude that a correct prediction of the system stability requires taking into account the coupled thermoacoustic effects of the heat source and sink.

The Rijke Tube Model

An illustration of the geometry and related dimensions of the Rijke tube and their corresponding values are available in Fig. 1 and Table 1, respectively. The tube is open at both ends and the flow direction implies the upstream and downstream parts. However, we do not include flow in the model. The medium is air and its temperature increases from $T_1$ to $T_2$ across the heat source and back to $T_1$ across the heat sink.

![Fig. 1. Geometry and dimensions of the Rijke tube (corresponding values are available in Table 1). The "flow" sign shows the assumed flow direction. We do not model the actual flow.](image)

We use the numerical Helmholtz solver COMSOL Multiphysics® [22] to calculate the complex eigenfrequencies of the system in frequency domain. The model is developed by Hoeijmakers et al. [23] and describes the propagation of small perturbations in an inhomogeneous medium including heat release. As described in [23], the inhomogeneous Helmholtz equation takes the form in Eq. (1) describing how fluctuating heat release/absorption acts as an acoustic monopole source,

\[
\frac{\omega^2}{\gamma p_0} p' + \nabla \left( \frac{1}{\rho} \nabla p' \right) = F \delta(x - L_1) \frac{\theta}{\rho} \frac{\partial p'(L_1)}{\partial x}
\]

(1)

where $p'$, $p_0$, $\rho$ and $\gamma$ are the fluctuating pressure (Pa), mean pressure (Pa), mean density (kg/m$^3$) and the ratio of specific heats, respectively. $\delta(x - L_1)$ is the Dirac delta function, $\theta = (T_1 - T_2)/T$ is the normalized temperature and $F$ is a transfer function for coupling the fluctuating heat release/absorption ($\dot{Q}$) to velocity ($u$) perturbations as

\[
F = \frac{\dot{Q} / \dot{Q}}{u / \dot{u}},
\]

where the prime and overbar denote the fluctuating and mean values, respectively. We use the $n$-$\tau$ (interaction index and time delay) formulation to model the transfer function as

\[
F = ne^{-\tau}. \quad (3)
\]

By numerically solving the constructed eigenvalue problem, one can obtain the complex eigenfrequency of the system showing the real eigenfrequency and its corresponding growth rate.

In this article, we consider only the smallest eigenfrequency of the system, i.e. the first acoustic mode, since it is close to the dominant resonant frequency in many thermoacoustic systems. However, a detailed analysis of higher modes of the system revealed complicated mode crossing and frequency loci veering. This is however outside the scope of this article. In addition, we assume acoustically compact heating and cooling elements of 5mm thickness, which is orders of magnitude smaller than the smallest acoustic wavelength in this configuration. Table 1 includes a list of the variables creating the parameter space. We present the results for some selected values between the mentioned minima and maxima. These are close to the experimental data available in the literature [16, 24]. The subscripts 1 and 2 denote the properties of the heat source and sink, respectively.

**Table 1. Geometrical and thermoacoustic parameters of the Rijke tube and the heat source (subscript 1) and sink (subscript 2).**

<table>
<thead>
<tr>
<th>parameter</th>
<th>fixed value</th>
<th>minimum value</th>
<th>maximum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>D (m)</td>
<td>0.05</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>L (m)</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$X_1$ (m)</td>
<td>0.25</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$X_2$ (m)</td>
<td>-</td>
<td>0.4</td>
<td>0.9</td>
</tr>
<tr>
<td>$L_2$ (m)</td>
<td>-</td>
<td>0.01</td>
<td>0.06</td>
</tr>
<tr>
<td>$T_1$ (K)</td>
<td>300</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$T_2$ (K)</td>
<td>-</td>
<td>350</td>
<td>1800</td>
</tr>
<tr>
<td>$n_1$</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$n_2$</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\tau_1$ (ms)</td>
<td>-</td>
<td>0.1</td>
<td>5</td>
</tr>
<tr>
<td>$\tau_2$ (ms)</td>
<td>-</td>
<td>0.1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Results and Discussion**

**Single-element heat exchanger – effects of temperature jumps and time delays**

The eigenfrequencies and corresponding growth rates of the system for various time delays of the source ($\tau_1$) and sink ($\tau_2$), and flame downstream temperature ($T_2$) of 350, 500 and 1800K are plotted in Fig. 2a, 2b and 2c, respectively. In each figure, the source and sink time delays vary from zero to 5 and 1 ms, respectively. This is depicted such that each pack of round markers connected with a black solid line, represents a specific source time delay (written next to it) and the sink time delay increases in the direction shown by the arrow. The grey filled triangles show the data for the cases with no heat sink, thus only one temperature jump and time delay in the system.
It is shown in Fig. 2a that including the heat exchanger decreases the eigenfrequency (the shift from grey triangles towards the round markers) due to introducing a cold region with smaller speed of sound. The growth rate, however, remains almost the same. For the small temperature jump in Fig. 2a, increasing the source time delay from zero destabilizes the system, but if the delay is large enough, the system is stable again. This happens at a source time delay of around 5.4ms irrespective of the sink time delay (the data at this time delay are not shown in the graph to avoid crowdedness). This period corresponds to the acoustic time scale at the frequency of 185Hz. Therefore, if the source time delay is large enough, the system behavior repeats itself and creates the self-similar pattern in Fig. 2a. It is worth noting that for these conditions, increasing the sink time delay monotonically destabilizes the system in the form of increases the growth rate to multiple times its value without a sink. All the eigenfrequencies are in a small frequency span of 14Hz and negligible frequency shift happens by changing the time delays of the source and sink.

Figure 2b shows the complex eigenfrequencies for a larger temperature jump, i.e. T2=500K. Here the shift to lower frequencies by addition of the heat exchanger is more pronounced, but gets weaker as the source time delay increases. The influence of increasing \( \tau_2 \) also weakens as \( \tau_1 \) increases. By increasing \( \tau_2 \), the system becomes unstable with its maximum growth rate around \( \tau_1=1.5 \text{ms} \), and stabilizes again after \( \tau_1=3 \text{ms} \). Compared to the case with small temperature jump (Fig. 2a), the eigenfrequency shift to lower frequencies by increasing \( \tau_1 \) is more significant here. However, the system behavior does not repeat itself with further increase in \( \tau_1 \) and the effects of \( \tau_2 \) fade out. This can be seen as the circles get more and more packed together as \( \tau_1 \) increases. In addition, the frequencies seem to converge to the intrinsic instability frequency of the system (this was not the case for the small temperature jump in Fig. 2a). It is well known from previous studies [25, 26] that the intrinsic instability frequencies correspond to the frequencies at which the phase of the flame transfer function equals odd multiples of \( \pi \). Therefore, for a constant time delay model we have,
\[
2\pi f = \frac{\phi}{\tau} = \frac{(2m+1)\pi}{\tau} \Rightarrow f = \frac{2m+1}{2\tau}
\]
(4)
where \( f, \phi \) and \( \tau \) are the frequency, phase and time delay of the flame transfer function. For the first intrinsic frequency (\( m=0 \)) we get \( f = 1/2\tau \), which yields values close to the frequencies plotted in Fig. 2b, when \( \tau_1 \geq 4 \text{ms} \). As the system mode gets close to the intrinsic instability frequency, the existence and time delay of the sink become less effective. This is a known characteristic of intrinsic thermoacoustic instabilities. We have investigated this phenomenon in more detail, but it is outside the scope of this article and will be the subject of a separate study. In Fig. 2b and when \( \tau_1 \leq 2 \text{ms} \), increasing \( \tau_2 \) first increases and then decreases the growth rate. If \( \tau_1 > 2 \text{ms} \), increasing \( \tau_2 \) monotonically increases the growth rate, similar to small temperature jumps in Fig. 2a. Depending on the value of \( \tau_1 \), the effects of increasing \( \tau_2 \) on the eigenfrequency can change from monotonic decrease (\( \tau_2 \leq 2 \text{ms} \)) to decrease-increase (\( 2.5 \leq \tau_2 \leq 3.5 \text{ms} \)) and monotonic increase (\( \tau_2 \geq 4 \)). In other words, the thermoacoustic behavior of the heat sink is not the same when placed downstream different heat sources. This two-way interaction is important because it signifies the importance of simultaneously designing the thermoacoustic properties of the burner and heat exchanger in heating appliances.

Fig. 2. Complex eigenfrequencies of the system when \( \tau_1=\{0:0.5:5\} \text{ms} \), \( \tau_2=\{0:0.1:1\} \text{ms} \), T1=300K and T2=350 (a), 500 (b) and 1800K (c). Each pack of round markers connected with a black solid line represent a specific \( \tau_1 \) (written next to it) and \( \tau_2 \) increases in the direction shown by the arrows. Triangles show cases with no heat sink, i.e. only one temperature jump and varying \( \tau_1 \).
The complex eigenfrequencies for the largest temperature jump studied ($T_2=1800K$) are shown in Fig. 2c. This case is mostly important because it is representative of idealized premixed combustion systems, which have been vastly studied experimentally, theoretically and numerically. Here the system is so unstable that increasing the source time delay cannot fully stabilize it and the growth rate remains positive. This is because the effects of the flame transfer function is much larger for large temperature jumps, which greatly amplifies the system instability. The effects of increasing sink delay is similar to the case with $T_2=500K$ in Fig 2b. When $\tau_1 \leq 2 ms$, the eigenfrequency reduces monotonically, but the growth rate changes non-monotonically. The situation is the reverse when $\tau_1 \geq 2.5 ms$, with the growth rate increasing monotonically and eigenfrequency non-monotonically. We can observe the convergence to intrinsic instability frequency with increasing source delay for this case as well.

It is also important to note that the system instability is most sensitive to sink delay when this delay is the smallest. This is visible in the form of the large vertical distance between the first two points in each pack of points in Fig. 2c. For example, when $\tau_1 = 1.5 ms$, the growth rate increases by 37% if $\tau_1$ increases from 0.0 to 0.1ms (this increase is around 13% if $\tau_1$ increases from 0.1 to 0.2ms). Since the time delay of the heat exchanger is associated to the hydrodynamic boundary layers and is relatively small [11], this finding motivates a detailed sensitivity analysis for experimentally measured values in a practical system.

**Multiple-element heat exchanger – effects of location and compactness**

The presented results on the single-element heat exchanger revealed that including the effects of the heat exchanger as an active acoustic element considerably changes the behavior of the complete system. Until now, we have chosen similar length scales for the heat source (burner) and heat sink (heat exchanger). However, the practical length scales of the heat exchangers can be larger than the burners. This motivates the next step towards a more realistic system, i.e. a system with distributed and multiple heat sinks. Here we consider the same heat source, but include 11 small heat sinks with a distancing of $L_2=0.01m$, to mimic a pinned heat exchanger. The location of the center of the pack of sinks changes from $X_2=0.4$ (close to the source) to 0.9m (close to the outlet). In addition, at $X_2=0.65m$, the distance between the sink elements increases from $L_2=0.01$ to 0.06m, in order to simulate the effects of distributed heat absorption. For the case with $L_2=0.06m$, the heat exchanger almost fills up the space downstream the heat source. The illustrations of these cases are presented in Fig. 3. The shades show the temperature change between 300K (white) and 1800K (black). In practical systems, the heat absorption through the heat exchanger occurs nonlinearly. However, here we assume that the temperature reduction occurs in multiple steps with equal temperature difference. This way we can more easily discover the effects of the distribution of the heat sinks. Therefore, the outlet temperature and thus the overall efficiency of the heat exchanger is equal in all the cases.

<table>
<thead>
<tr>
<th>$X_2=0.40, L_2=0.01$</th>
<th>$X_2=0.45, L_2=0.01$</th>
<th>$X_2=0.50, L_2=0.01$</th>
<th>$X_2=0.55, L_2=0.01$</th>
<th>$X_2=0.60, L_2=0.01$</th>
<th>$X_2=0.65, L_2=0.01$</th>
<th>$X_2=0.65, L_2=0.02$</th>
<th>$X_2=0.65, L_2=0.03$</th>
<th>$X_2=0.65, L_2=0.04$</th>
<th>$X_2=0.65, L_2=0.05$</th>
<th>$X_2=0.65, L_2=0.06$</th>
<th>$X_2=0.70, L_2=0.01$</th>
<th>$X_2=0.75, L_2=0.01$</th>
<th>$X_2=0.80, L_2=0.01$</th>
<th>$X_2=0.85, L_2=0.01$</th>
<th>$X_2=0.90, L_2=0.01$</th>
</tr>
</thead>
</table>

Fig. 3. Test cases for multiple-element and distributed heat exchanger. The location of the center of the 11-element heat exchanger varies from 0.4 to 0.9m and the distance between the elements varies from 0.01 to 0.06m. The shades show the temperature change between 300K (white) and 1800K (black).

The interaction index of the source and sinks are set to unity and the sink time delay is fixed at 1ms, while the source time delay varies from 1 to 5ms. As discussed in the single-element section, the source time delay is important since it can alter the stability trends when the sink delay changes. The corresponding complex eigenfrequencies of the system are plotted in Fig. 4. The black and grey arrows show the direction of increasing $X_2$ (distance between the source and sinks) and $L_2$ (distance between the sink elements), respectively.

<table>
<thead>
<tr>
<th>$\tau_1=1$</th>
<th>$\tau_1=2$</th>
<th>$\tau_1=3$</th>
<th>$\tau_1=4$</th>
<th>$X_2=0.4$</th>
<th>$X_2=0.01$ to 0.06</th>
</tr>
</thead>
</table>

Fig. 4. The complex eigenfrequencies of the system for $T_2=300K$, $T_2=1800K$, $n=1$, $n=[1:5]$ms, $n=1$ms, $X_2=0.4, 0.05, 0.09$ (and $L_2=0.01:0.01:0.06$). Each pack of round markers connected with a black solid line represent a specific $\tau_1$ (written next to it) and the black arrow shows the direction of increasing $X_2$. At $X_2=0.65m$, $L_2$ increases in the direction of the grey arrows, connecting the grey triangles.
The trend of increasing and decreasing growth rate with increasing source time delay is also present here, similar to the single-element sink. The values of growth rates are also in the same order of magnitude (see Fig. 2c). However, the location of the sink has significant effects on the system stability, especially for small source time delays. This is intuitive, since a heat sink with the same temperature jump and time delay as a heat source can be the most effective. Increasing the distance between the source and sink results in the growth rate reaching a maximum and then decreasing. The location at which this maximum growth occurs, changes from very close to the outlet when $\tau_1=1\text{ms}$ to very close to the heat source when $\tau_1=5\text{ms}$. A source time delay close to practical premixed perforated burners is around 3ms. The $\tau_1=3\text{ms}$ curve in Fig. 4 shows that increasing the distance between the heat source and a pack of heat sinks with 1ms time delay can increase the growth rate of the system by 17% or decrease it by 24%. These values are significantly important in predicting the stability of the system and need to be taken into account in the design process. Therefore, the distance between the burner and the heat exchanger is not only affecting the emissions, but also the thermoacoustics and system stability.

Finally, Fig. 4 also shows that spreading the heat sink elements has negligible effects on stability when the source delay is small ($\tau_1=1\text{ms}$), but it decreases the growth rates towards more stable conditions if the source time delay is larger ($\tau_1\geq2\text{ms}$). This means that a distributed heat exchanger design is more favorable for preventing thermoacoustic instabilities.

Conclusions

We studied a Rijke tube containing a heat source and multiple heat sinks as a simplified, but representative, model to investigate the thermoacoustic instabilities in heating appliances. Studying the effects of several parameters helped understanding the thermoacoustic behavior of such systems in various operating conditions. Among the most important parameters are the temperature jump and time delay of the heat source, which are affected by the choice of the combustion properties of the burner. In addition, the distance between the heat source and sink is a key parameter in designing the emission properties of heating appliances.

The results showed that various situations of stability and instability can occur, which depend on the properties of both the heat source and sink. The effects showed a large shift in the eigenfrequency and growth rate as well. When the source and sink are placed at the first and third quarter of the tube, respectively, and the source delay is small, increasing the sink delay first increases and then decreases the growth rate of the system. However, for larger values of source delay close to practical systems, increasing the time delay of the heat exchanger almost monotonically increases the growth rates. This finding is not trivial and means that the source time delay can alter the stability trends when sink time delay changes. We also observed that if the temperature jumps are large enough, increasing the source time delay converges the first system mode to its intrinsic instability mode. When the system mode is near an intrinsic mode, the effects of most parameters fade out due to the nature of the intrinsic modes. Studying the effects of distributed heat sinks revealed that there is a critical location for the heat sink, which results in the maximum growth rate. In addition, we found that spreading the sink elements throughout the system reduces the growth rate.

The effects observed in this study are in some configurations to the extent that the system is predicted unstable or stable, depending on whether or not the heat exchanger is included. Therefore, it is crucial to include the active thermoacoustic effects of the heat exchanger for correct predictions of system stability. An extension of the model used in this work can be developed to predict instabilities in complex full-scale heating appliances.

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References