Theoretical determination of flame transfer function using the G-equation – Comparison with experiments

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Motivations

• One of the major problems encountered while designing rocket motors, jet engines, ground gas turbines, industrial furnaces,…

• Characterised by large flow oscillations

Flashback → Extinction
Heat Flux Increase → Radiated Noise
Structural Vibrations
Consequences...

Example of a combustion instability leading to flashback
Research on instabilities

- Understanding of the fundamental mechanisms of dynamic interactions
- Description of the instabilities in complex geometries
- Developments of passive and active control methods
Simplistically…

\[ \Delta u \rightarrow \Delta Q \] acoustic wave (induced) forcing

\[ \Delta Q \rightarrow \Delta p \] combustion noise

\[ \Delta u, \Delta p \]: acoustic waves

instability \( \leftrightarrow \) phase match
Contents

➢ Flame response to acoustic modulations
  ➢ Simple model
  ➢ Experiments
➢ G-equation calculations
➢ Velocity field analysis
➢ Conclusions and perspectives
Perturbation equation

Assumptions:
- Low speed reactive flows $d./dt \sim \partial./\partial t$
- $\gamma$ constant
- Weak pressure waves: $p = p_0 + p_1$ with $p_1 \ll p_0$
- Mean pressure does not change spatially

Then ($\ldots$)

$$\nabla \cdot c^2 \nabla p_1 - \frac{\partial^2 p_1}{\partial t^2} = \frac{\partial}{\partial t} \left[ (\gamma - 1) \sum_{k=1}^{N} h_k \dot{\omega}_k \right] - \gamma p_0 \nabla \mathbf{v} : \nabla \mathbf{v}$$

Wave-like equation
Heat release source-term

- Assuming a single reaction step and equal $c_{pk}$

$$\frac{\partial}{\partial t} \left[ (\gamma - 1) \sum_{k=1}^{N} h_k \dot{\omega}_k \right] = - \frac{\partial}{\partial t} \left[ (\gamma - 1)(-\Delta h_f^0) \dot{\omega} \right]$$

$\Delta h_f^0$: formation enthalpy per unit mass of the mixture
$\dot{\omega}$: rate of reaction

$$-(\gamma - 1)(-\Delta h_f^0) \frac{\partial \dot{\omega}}{\partial t} \quad \text{or} \quad -(\gamma - 1) \frac{\partial q_1}{\partial t}$$

$q_1$: nonsteady rate of heat release per unit mass of fuel
Acoustic Energy Budget

\[ e = \frac{1}{2} \frac{(p')^2}{\rho_0 c_0^2} + \frac{1}{2} \rho_0 (u')^2 \]

- Dissipation terms neglected
- Low Mach number

\[ \left\langle \frac{\partial e}{\partial t} \right\rangle_T = \left\langle \int_V \frac{\gamma - 1}{\gamma \rho_0} p' q' dV \right\rangle_T - \left\langle \int_{\Sigma} p' u' \cdot nd\Sigma \right\rangle_T \]

Rayleigh source term

Acoustic fluxes at boundaries

\[ S \]

\[ \Phi_{ac}^i \]
For premixed systems

Adapted from Paschereit et al. (1998)
Driving processes

Unsteady strained flames

\[ \phi = \phi_0 + \phi' \]

Equivalence ratio perturbations

Flame vortex interactions

Response to equivalence ratio modulations
Driving processes (2)

Acoustically modulated flames

Perturbed flames interacting with a wall

studied in the following


Ducruix et al., Prog. in Astronautics and Aeronautics (2005)
Dynamics of instabilities

Typical instability mechanism

Flow → Combustion

Feedback

Instability modelling

Flow solver → Combustion model

Acoustic resolution
Typical instability mechanism

Flame Transfer Function (FTF)

Flow solver → Combustion model

Acoustic Forcing

Linear assumption or Flame Describing Function (FDF) framework
\[ \Delta u \rightarrow \Delta Q \text{ acoustic (induced) forcing} \]

\[ \Delta Q \rightarrow \Delta p \text{ combustion noise} \]

\[ (\Delta u, \Delta p): \text{acoustic waves} \]

instability \leftrightarrow \text{phase match}
Model for laminar cases

Experimental determination of the flame response (wide frequency range and modern diagnostics)

Understanding and modelling of the interaction phenomena based on Fleifil et al. (96)

↑↑↑↑↑↑ acoustic modulation
Modelling of the flame response

Aim:  Transfer function: \( \frac{Q_1}{Q_0} = f \left( \frac{v_1}{v_0} \right) \)

Assumptions

- Constant flame burning velocity \( S_L \)
- Axial velocity field in the fresh gases
- Velocity field spatially uniform in the fresh gases

Ducruix et al. (2000)
Simple model

Starting from the G-equation:
\[ \frac{\partial G}{\partial t} + \mathbf{v} \cdot \nabla G = -S_L |\nabla G| \]

\( \mathbf{v} \): velocity vector, \( S_L \): (laminar) flame displacement speed

Introducing \( \eta \) as \( G = \eta - y \), one gets
\[ \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial r} - v = -S_L \left[ 1 + \left( \frac{\partial \eta}{\partial r} \right)^2 \right]^{1/2} \]

Assuming \( \eta = \eta_0 + \eta_1 \) and \( \eta_1 << \eta_0 \)

\[ \frac{\partial \eta_1}{\partial t} = S_L \cos \alpha_0 \frac{\partial \eta_1}{\partial r} + v_1 \]
Simple model (2)

\[ \frac{\partial \eta_1}{\partial t} = S_L \cos \alpha_0 \frac{\partial \eta_1}{\partial r} + v_1 \]

Area fluctuations
\[ A_1 = 2\pi \cos \alpha_0 \int_0^R \eta_1 \, dr \]

Heat release fluctuations
\[ Q_1 = \rho_U S_L \Delta q A_1 \]

Then (…)
\[ \frac{Q_1}{Q_0} = \frac{v_1}{v_0} \frac{2}{\omega_*^2} \left[ (1 - \cos \omega_*) \cos \omega t + (\omega_* - \sin \omega_*) \sin \omega t \right] \]

Relevant parameter: reduced frequency
\[ \omega_* = \frac{\omega R}{S_L \cos \alpha_0} \]
Transfer function amplitude

\[
|F(\omega_*)| = \frac{2 \left[ (1 - \cos \omega_*)^2 + (\omega_* - \sin \omega_*)^2 \right]^{1/2}}{\omega_*^2}
\]

\[
|H(\omega_*)| = \frac{\beta}{\left( \beta^2 + \omega_*^2 \right)^{1/2}}
\]

(First order system \( \beta = 3 \))
Transfer function phase

\[ \phi(\omega_*) = \tan^{-1}\frac{\omega_* - \sin \omega_*}{1 - \cos \omega_*} \]

\[ \psi(\omega_*) = \tan^{-1}\frac{\omega_*}{\beta} \]

(First order system \( \beta=3 \))
Experimental configuration

- Simplified but perfectly controlled configuration
- Wide range of frequencies and amplitudes

Diagnostics: Schlieren technique, L.D.V. Spontaneous emission
Schlieren visualisations

\[ \Phi = 0.95, \ \nu_0 = 0.96 \text{ m.s}^{-1}, \ \omega_* = 5 \]

methane-air flame
$\Phi = 0.95$, $v_\theta = 0.96 \text{ m.s}^{-1}$, $\omega_\star = 15$

methane-air flame
Schlieren visualisations

\[ \Phi = 0.95, \, v_0 = 0.96 \, \text{m.s}^{-1}, \, \omega_\ast = 5 \]

methane-air flame
Schlieren visualisations

$\Phi = 0.95, \nu_0 = 0.96 \text{ m.s}^{-1}, \omega_* = 15$

methane-air flame
Experimental measurements

\( \frac{v_1}{v_0} : 8 - 20 \% \)
\( f_{\text{mod}} : 5 - 300 \text{ Hz} \)

\( \omega_* : 1 - 60 \)  
(Ø22 mm)

\( \omega_* : 1.7 - 100 \)  
(Ø30 mm)

Whatever the modulation conditions

- Almost sinusoidal signals of velocity and emission
- Main peak @ modulation frequency (negligible harmonics)
Transfer function amplitude

\[ (\frac{I_1}{I_0})(\frac{v_0}{v_1}) \]

\[ \omega^* \]

\[ V_0=0.96 \text{m/s} \]
\[ V_0=1.20 \text{m/s} \]
\[ V_0=1.44 \text{m/s} \]
\[ V_0=1.68 \text{m/s} \]

\( |F(\omega^*)| \)
\( |H(\omega^*)| \)

(\( \varnothing \)22 mm)
Transfer function phase

- $\phi (\omega^*)$
- $\psi (\omega^*)$

- $V_o=0.96$ m/s
- $V_o=1.20$ m/s
- $V_o=1.44$ m/s
- $V_o=1.68$ m/s

Phase difference (rad)

$\omega^*$

$(\phi 22 \text{ mm})$
Transfer function analysis

- Good modelling of the flame behaviour for (very) low frequencies $\omega_* < 6$ for the amplitude and $\omega_* < 2$ for the phase
  BUT underestimation for intermediate frequencies
  May be due to (too) strong assumptions

- Check the analytical solution of the flame transfer function
  ✴ G-equation calculations of the flame response
  ✴ Level set approach to handle strong deformations
G-Equation Calculation

• G-Equation

\[ \frac{\partial G}{\partial t} + \mathbf{v} \cdot \nabla G = -S_D |\nabla G| \]

• Resolution based on the flux splitting principle. First, non linear propagation then linear advection mechanism

• Calculations
  ➢ Level set approach
  ➢ Coarse grid: 41 × 51 points
  ➢ Schemes
    • Time \( \text{RK2, RK3} \)
    • Propagation (H-J) \( \text{WENO3-5} \)
    • Advection (hyp) \( \text{WENO3-5} \)
Simple model simulation

\[ v = \bar{v} + a \cos(\omega t) \]
\[ u = 0 \]
\[ \frac{a}{\bar{v}} = 0.2 \]

Failure!
Transfer function analysis

➢ Discrepancies may be due to strong assumptions on velocity.
   ✴ PIV measurements
   ✴ Realistic description of velocity fields

➢ No analytical solution of the flame transfer function
   ✴ $G$-equation calculations of the flame response
   ✴ Necessary to propose a realistic modelling of the velocity in the fresh gases
Velocity field, $\omega_* = 2$

- Small axial gradient, small radial velocity.
- Validation of the assumptions on the velocity field
- Trends of the transfer function correctly reproduced
Velocity field, $\omega_\ast = 15$

- Large axial gradient, large radial velocity
- Too strong assumptions on the velocity field
- Bad representation of the transfer function
Key idea: phase difference $\varphi$ between velocity and acoustic modulation depends on $y$ (see De Soete, 1964, Baillot et al., 1998).

> Assumption: 
$\varphi(y) = -ky + b$

> Determination of $k$
- Experimentally
- Using:
  $k \approx K = \frac{\omega}{V_0}$

Phase difference = convection of perturbations by the flow
Velocity modelling (2)

1. Determination of axial velocity characteristics $V_0$ and $v'_{max}$
   
   \[ V = V_0 + v'_{max} \cos (\omega t - \varphi) \]

2. Estimation of the phase difference $\varphi$
   
   \[ \varphi = k y \quad \text{using} \quad k = \omega / V_0 \]

3. Determination of radial velocity using $\nabla \cdot \mathbf{V} = 0$
   
   \[ U = \frac{1}{2} k v'_{max} \sin (\omega t - \varphi) \]

Remarks:

➢ Variations of $V_0$ and $v'_{max}$ with $y$ are not taken into account
➢ Seems to be relevant for a certain range of frequencies

Schuller et al. (2003)
Axial velocity modelling

Experimental results

Corresponding modelling

\( \omega_* = 10 \), representation at a given phase

Good estimation of the evolution of the axial velocity with \( y \)
Radial velocity modelling

Experimental results

Corresponding modelling

\( \omega_* = 10 \), representation at the same given phase

Good estimation of the evolution of the radial velocity
Simple model simulation

\[ v = \bar{v} + a \cos(\omega t) \]
\[ u = 0 \]
\[ \frac{a}{\bar{v}} = 0.2 \]

Failure!
\[ v = \bar{v} + a \cos(ky - \omega t) \]
\[ u = \frac{1}{2} k(x - 20)a \sin(ky - \omega t) \]
\[ \frac{a}{\bar{v}} = 0.2 \]
\[ \omega = k\bar{v} \]

Much better!
FTF modeling

GAIN

PHASE

\[ \frac{Q_{\text{rms}}}{V} \]

\[ \omega_* \]

\[ \text{phase difference (rad)} \]

\[ \text{ER} = 1.05 \quad S_L = 0.39 \text{ m/s} \quad V = 0.97 \text{ m/s} \quad v' = 0.19 \text{ m/s} \]

Much better!
Conclusions

- Possibility to study other interaction modes
  - Instabilities in LPP burners
  - Tangential modes in rocket engines

- Methods useful to study quasi-laminar industrial burners
  - Simplified modelling tool
  - Helpful for the design of burners

- Interesting configuration for the validation of calculation codes
  - Completely controlled situation
  - Capability of a CFD code to simulate interactions